**2. Two equal and fair dice are rolled, and we observed two numbers X and Y.**

**1. Find RX, RY, and the PMFs of X and Y.**

**2. Find P(X = 2,Y = 6).**

**3. Find P(X&gt;3|Y = 2).**

**4. If Z = X + Y. Find the range and PMF of Z.**

**5. Find P(X = 4|Z = 8).**

Sure, let’s solve these step by step:

1. **RX, RY, and the PMFs of X and Y:**
   * The range of X and Y (RX and RY) is {1, 2, 3, 4, 5, 6} because a die has 6 faces numbered from 1 to 6.
   * The probability mass function (PMF) of X and Y is the same, given by P(X=x) = P(Y=y) = 1/6 for x, y ∈ {1, 2, 3, 4, 5, 6}. This is because the dice are fair.
2. **P(X = 2,Y = 6):**
   * The events “X = 2” and “Y = 6” are independent. So, P(X = 2, Y = 6) = P(X = 2) \* P(Y = 6) = (1/6) \* (1/6) = 1/36.
3. **P(X>3|Y = 2):**
   * Given that Y = 2, the event “X > 3” corresponds to X taking values in {4, 5, 6}. Since X and Y are independent, P(X > 3 | Y = 2) = P(X > 3) = P(X=4) + P(X=5) + P(X=6) = (1/6) + (1/6) + (1/6) = 1/2.
4. **Range and PMF of Z:**
   * Z = X + Y can take values in the range {2, 3, …, 12}.
   * The PMF of Z is given by P(Z=z) = Σ P(X=x, Y=z-x) for each (x, z-x) such that x, z-x ∈ {1, 2, …, 6} and z = x + (z - x). For example, P(Z=2) = P(X=1,Y=1) = (1/36), P(Z=3) = P(X=1,Y=2) + P(X=2,Y=1) = 2/36, and so on.
5. **P(X = 4|Z = 8):**
   * Given Z = 8, the event “X = 4” corresponds to Y taking values in {4}. So, P(X = 4 | Z = 8) = P(X = 4, Y = 4) / P(Z = 8). You can calculate P(Z = 8) using the PMF of Z derived above, and P(X = 4, Y = 4) = (1/36).

Please note that these solutions assume that the two dice rolls are independent events. If this is not the case, the solutions may vary.

**3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible**

**options. A student knew the answer to 10 questions, but the other 10 questions were unknown**

**to him, and he chose answers randomly. If the student X’s score is equal to the total number**

**of correct answers, then find out the PMF of X. What is P(X>15)?**

Let’s denote the number of questions the student knows the answer to as K and the number of questions the student guesses as G. In this case, K=10 and G=10.

The student will definitely answer all K questions correctly. For the G questions, the student guesses randomly among 4 options, so the probability of guessing correctly is 1/4.

Let’s denote Y as the number of correct answers among the G questions. Y follows a binomial distribution with parameters n=G and p=1/4.

Therefore, the total score X is the sum of the correct answers from the known and guessed questions, i.e., X = K + Y.

The probability mass function (PMF) of X is then the PMF of Y shifted by K, which is a binomial distribution with parameters n=G and p=1/4, shifted by K.

The probability P(X>15) is calculated as 1 - P(X<=15), where P(X<=15) is the cumulative distribution function (CDF) of X at 15.

In mathematical terms:

* PMF of X: P(X=k) = C(G, k-K) \* (1/4)^(k-K) \* (3/4)^(G-k+K) for k in {K, K+1, ..., K+G}
* P(X>15) = 1 - Σ[P(X=k)] for k in {10, 11, ..., 15}

Please note that C(n, k) denotes the binomial coefficient, which is the number of ways to choose k successes out of n trials.

You can calculate these probabilities using statistical software or programming languages that support statistical functions. For example, in Python, you can use the scipy.stats module to calculate binomial probabilities.

Here’s a Python code snippet that calculates P(X>15):

from scipy.stats import binom

K = 10

G = 10

p = 1/4

# Calculate P(X<=15)

p\_le\_15 = sum(binom.pmf(k, G, p) for k in range(K, 16))

# Calculate P(X>15)

p\_gt\_15 = 1 - p\_le\_15

print(f'P(X>15) = {p\_gt\_15}')

This code calculates the binomial probabilities for X from 10 to 15, sums them up to get P(X<=15), and then subtracts this from 1 to get P(X>15). The result is printed out. Please replace the print statement with your preferred method of output if necessary.

Remember to run this code in a Python environment where the scipy library is installed. If it’s not installed, you can install it using pip: pip install scipy.

Please note that this is a theoretical calculation based on the assumptions stated. In real-world situations, other factors may affect the outcome.

**4. The number of students arriving at a college between a time interval is a Poisson random**

**variable. On average, 10 students arrive per hour. Let Y be the number of students arriving**

**from 10 am to 11:30 am. What is P (10<Y≤15) ?**

The number of students arriving in a given time interval follows a Poisson distribution. The probability mass function (PMF) of a Poisson distribution is given by:

\[ P(Y = k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!} \]

where:

- \( Y \) is the random variable representing the number of arrivals,

- \( \lambda \) is the average rate of arrivals per unit time (in this case, per hour),

- \( k \) is the number of arrivals.

Given that the average rate (\( \lambda \)) is 10 students per hour, and the time interval is from 10 am to 11:30 am (1.5 hours), the expected number of arrivals during this interval (\( \lambda\_{\text{interval}} \)) is \( \lambda\_{\text{interval}} = \lambda \times \text{interval duration} \).

\[ \lambda\_{\text{interval}} = 10 \times 1.5 = 15 \]

Now, we can calculate the probability \( P(10 < Y \leq 15) \) using the Poisson PMF:

\[ P(10 < Y \leq 15) = \sum\_{k=11}^{15} P(Y = k) \]

\[ P(10 < Y \leq 15) = \sum\_{k=11}^{15} \frac{e^{-15} \cdot 15^k}{k!} \]

Calculate this sum to find the desired probability. Note that the sum involves adding the probabilities for \( k = 11, 12, 13, 14, 15 \).

**5.Two independent random variables, X and Y, are given such that X~Poisson(α) and**

**Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z .**

The sum of two independent Poisson-distributed random variables is itself a Poisson-distributed random variable. If \(X \sim \text{Poisson}(\alpha)\) and \(Y \sim \text{Poisson}(\beta)\), then the sum \(Z = X + Y\) follows a Poisson distribution with parameter \(\lambda = \alpha + \beta\).

The probability mass function (PMF) of a Poisson distribution is given by:

\[ P(Z = z) = \frac{e^{-\lambda} \cdot \lambda^z}{z!} \]

where:

- \( Z \) is the random variable representing the sum of the Poisson-distributed variables,

- \( \lambda \) is the parameter of the Poisson distribution, which is the sum of the parameters of \(X\) and \(Y\), i.e., \( \lambda = \alpha + \beta \),

- \( z \) is the observed value for the sum.

Therefore, the PMF of \(Z = X + Y\) is:

\[ P(Z = z) = \frac{e^{-(\alpha + \beta)} \cdot (\alpha + \beta)^z}{z!} \]

This is the PMF for the sum of two independent Poisson-distributed random variables

**6.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.**

To find the joint probability mass function (PMF) of X (number of white shirts) and Y (number of black shirts) when 10 shirts are chosen without replacement from a box containing 40 white shirts and 60 black shirts, we can use the hypergeometric distribution.

The hypergeometric distribution is suitable for situations where the sampling is done without replacement from a finite population of items with two categories (success and failure).

The joint PMF of X and Y is given by:

\[ P(X = x, Y = y) = \frac{\binom{40}{x} \cdot \binom{60}{y}}{\binom{100}{10}} \]

where:

- \( \binom{n}{k} \) represents the binomial coefficient, which gives the number of ways to choose \(k\) items from a set of \(n\) items.

- \( x \) is the number of white shirts chosen.

- \( y \) is the number of black shirts chosen.

- \( \binom{40}{x} \) is the number of ways to choose \(x\) white shirts from the available 40 white shirts.

- \( \binom{60}{y} \) is the number of ways to choose \(y\) black shirts from the available 60 black shirts.

- \( \binom{100}{10} \) is the total number of ways to choose 10 shirts from the total pool of 100 shirts.

The constraints are \(0 \leq x \leq 10\) and \(0 \leq y \leq 10\), and \(x + y \leq 10\) (since we are choosing a total of 10 shirts).

Calculate \(P(X = x, Y = y)\) for all valid combinations of \(x\) and \(y\). The joint PMF represents the probability of choosing \(x\) white shirts and \(y\) black shirts out of the 10 shirts selected.

**8.There are 100 men on a ship. If Xi is the ith man’s weight on the ship and Xi’s are independent and**

**identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men’s total**

**weight on the ship exceeds 18,000.**

This is a problem of the sum of independent and identically distributed (i.i.d.) random variables, which can be solved using the Central Limit Theorem (CLT).

The total weight of the men on the ship is the sum of the weights of each man, which can be represented as:

S = X\_1 + X\_2 + ... + X\_{100}

whereX\_iis the weight of the $i$th man.

According to the Central Limit Theorem, for largen, the sumSofni.i.d. random variables, each with mean\muand standard deviation\sigma, is approximately normally distributed with meann\muand standard deviation\sqrt{n}\sigma.

In this case,n = 100,\mu = 170, and\sigma = 30. So, the mean ofSisn\mu = 100 \times 170 = 17000, and the standard deviation ofSis\sqrt{n}\sigma = \sqrt{100} \times 30 = 300.

We want to find the probability thatSexceeds 18000, i.e.,P(S > 18000). To do this, we standardizeSto a standard normal random variableZ:

Z = \frac{S - n\mu}{\sqrt{n}\sigma}

So, we want to findP(Z > \frac{18000 - n\mu}{\sqrt{n}\sigma}) = P(Z > \frac{18000 - 17000}{300}) = P(Z > \frac{1000}{300}) = P(Z > 3.33).

Using the standard normal distribution table, we know thatP(Z < 3.33) \approx 0.9996. Therefore,P(Z > 3.33) = 1 - P(Z < 3.33) = 1 - 0.9996 = 0.0004.

So, the probability that the total weight of the men on the ship exceeds 18000 is approximately 0.0004 or 0.04%. Please note that this is an approximation based on the Central Limit Theorem, and the actual probability may be slightly different.

**9.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF**

**If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.**

Given that \(X\_1, X\_2, \ldots, X\_{25}\) are independent and identically distributed random variables, and we want to estimate \(P(4 \leq Y \leq 6)\) where \(Y = X\_1 + X\_2 + \ldots + X\_{25}\), we can use the Central Limit Theorem (CLT) to approximate the distribution of the sum.

The CLT states that the sum (or average) of a large number of independent and identically distributed random variables will be approximately normally distributed, regardless of the original distribution of the variables.

Given that you have a probability mass function (PMF) for the individual random variables, we can calculate the mean (\(\mu\)) and standard deviation (\(\sigma\)) of the sum \(Y\) using the properties of the individual random variables.

Let \(p(k)\) be the PMF of \(X\_i\), where \(k\) is the value taken by \(X\_i\).

\[ \mu = E(X\_i) = \sum\_{k} k \cdot p(k) \]

\[ \sigma = \sqrt{Var(X\_i)} = \sqrt{\sum\_{k} (k - \mu)^2 \cdot p(k)} \]

Once you have the mean (\(\mu\)) and standard deviation (\(\sigma\)) of \(Y\), you can standardize the variable and use the standard normal distribution to estimate \(P(4 \leq Y \leq 6)\).

\[ Z = \frac{Y - \mu}{\sigma} \]

\[ P(4 \leq Y \leq 6) \approx P\left(\frac{4 - \mu}{\sigma} \leq Z \leq \frac{6 - \mu}{\sigma}\right) \]

However, to proceed further, we would need the specific values of \(p(k)\) for the PMF of \(X\_i\) or additional information about the distribution of \(X\_i\). If you can provide the PMF or more details about \(X\_i\), I can assist you in calculating \(\mu\), \(\sigma\), and estimating \(P(4 \leq Y \leq 6)\) using the CLT.